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superposition, residual solids will remain, or solids composed of residua, in either solid, which will not be superposed upon each other. Because since for example the figures $E'F'$, TH' are equal to the figure TV, then when the common figure TH' is taken away, the remaining figure $E'F'$ will be equal to the remaining figure $H'V$; and this will happen in any plane whatever parallel to PQ and meeting the solids ABC, XYZ. Therefore whenever we have a residuum of one solid, we shall always have a residuum of the other also; and it will be evident, according to the method applied in the former part of this Proposition in the case of plane figures, that the residua of the solids, or the aggregates of residua, will always be between the same parallel planes (as the residua $LB'YTF'$, $MC'Z$, Thg are between the same parallels DN, RS) and will be analogues.

Now if these residua be superposed again, so that the plane DL will be placed on the plane LN, and RY on YS, and this is understood to be done continually, until ABC, which is being superposed, is entirely taken, the entire solid ABC will finally coincide with the entire solid XYZ. For when the entire solid ABC is superposed upon XYZ, unless they coincided there would be some residuum of one, as of the solid XYZ, therefore also some residuum of the solid $XB'C'$, or ABC, and this residuum would not be superposed; which is absurd, for it is already assumed that the entire solid ABC is superposed on XYZ. Therefore there will not be any residuum in these solids; therefore they will coincide; therefore the solid figures spoken of, ABC, XYZ, will be equal to each other, which was to be proved of them.

MATHEMATICAL FORMS OF CERTAIN ERODED MOUNTAIN SIDES.

By T. M. PUTNAM, University of California.

In some of the desert regions of the west, visited by occasional heavy rainstorms, the formation is such that erosion takes place in a way that makes it possible to calculate approximately the form of the curve of intersection of the eroded slope and the alluvial fan of the plain.

This problem has been suggested by Professor A. C. Lawson of the department of geology of the University of California. He has formulated the hypotheses used below and the forms of the curves here obtained have been closely approximated by the actual geological conditions observed by him.

In the figure $S_0OR_0T_0$ is the contour of valley, hillside and plateau at some initial time. The table land R_0T_0 may be taken as inclined at a small angle C to the horizontal direction T_0Y. The valley OS_0 is assumed to be inclined at an angle B. The eroded material OR_0RH comes off in layers and is deposited in the valley in the position S_0SHO. The slope of erosion as well as the floor of the valley maintains, in the observed regions, a fairly constant inclination. Both inclinations are assumed constant in these calculations. The line S_0S, at the low

point in the valley, is taken vertical. An alluvial fan, similar to the one under discussion, would be deposited from the opposite slope also terminating at S_0S.

The problem is to find the locus of the point H, or in other words the form of

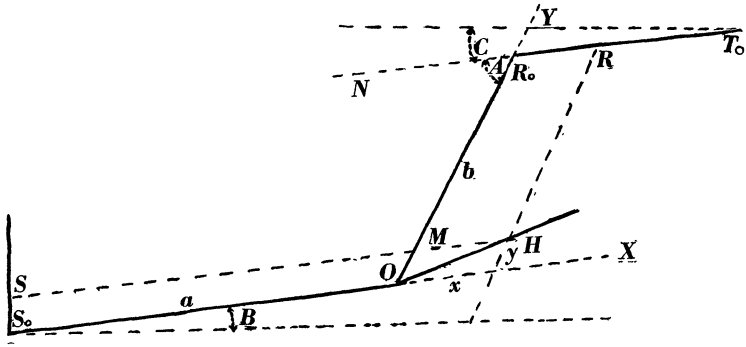


FIG. 1.

the uneroded portion of a cross-section. The geometrical conditions require that the area OR_0RH be equal to the area S_0SHO.

Choosing oblique coördinates as indicated in the figure and letting $OS_0 = a$, $OR_0 = b$ and $\angle\ OR_0N = A$, it follows that

$$MR_0RH = x(b - y)\sin(A + C - B) + \frac{1}{2}\frac{x^2 \sin(C - B)\sin(A + C - B)}{\sin A},$$

$$S_0SMO = ay\sin(A + C - B) + \frac{1}{2}\frac{y^2 \cos(A + C)\sin(A + C - B)}{\cos B}.$$

Equating these two expressions and letting

$$P = \frac{\sin(C - B)}{\sin A}, \qquad Q = \frac{\cos(A + C)}{\cos B}$$

one obtains the equation of the locus of H in the form

$$Px^2 - 2xy - Qy^2 + 2bx - 2ay = 0.$$

To investigate the form of this conic one considers the sign of $1 + PQ$. Conditions occurring in nature require $A + C > B$ hence $0 < Q < 1$. Since $A > B - C$ then when $B > C$ we have $|P| < 1$. Hence $1 + PQ > 0$. Therefore the conic is always a hyperbola in the cases under consideration.

The discriminant[1] of the equation is $b^2Q + 2ab - a^2P$ which vanishes when

$$\frac{a}{b} = \frac{1 + \sqrt{1 + PQ}}{P}.$$

Since a/b is positive this requires that $P > 0$ and hence $C > B$. Under this

[1] C. Smith, *Conic Sections* (1912 ed.), page 43.

condition there is for every set of angles A, B, C one ratio $a : b$ for which the locus is a straight line.

The tangent to the curve at O is parallel to the line S_0R_0 and at any point H it is parallel to the line SR. The curve is concave downward

$$\text{if } \frac{a}{b} < \frac{1+\sqrt{1+PQ}}{P}, \qquad \text{upward if } \frac{a}{b} > \frac{1+\sqrt{1+PQ}}{P}.$$

The special case $B = C = 0$ is of interest. The equation now becomes $\cos A y^2 + 2xy - 2bx + 2ay = 0$. Using rectangular coördinates, u and v, with the old origin and X-axis,

$$u = x + y \cos A, \qquad v = y \sin A,$$

the equation becomes

$$\cot A \cdot v^2 - 2uv + 2ku - 2hv = 0,$$

where $h = a + b \cos A$, $k = b \sin A$. The center is at the point $(-a, k)$ which is the point where S_0S meets the now horizontal line T_0R_0. One of the asymptotes is the line R_0T_0. The curve is concave downward in this case and the uneroded portion tends to become horizontal as erosion progresses. It might be expected that R_0T_0 is always an asymptote, but this is the case only when $\angle B = \angle C$, *i. e.*, $P = 0$.

THE OBSOLETE IN MATHEMATICS.

By G. A. MILLER, University of Illinois.

The present article was suggested by extravagant claims made in the advertisements of some general works of reference. As an extreme case we may refer to recent advertisements of the *Mathematical Dictionary and Cyclopedia of Mathematical Science* by Charles Davies and W. G. Peck, which contained the following statements: "A standard work for 60 years." "Definitions of all terms employed in mathematics, an analysis of each branch and the whole as forming a single science."

The claims represented by these quotations, which many readers of this MONTHLY have doubtless seen, present a serious affront to our intelligence, since any mathematical reader should at once recognize that these claims must be false even if he has never heard of the work in question. Hence it is not the object of the present article to establish their extravagance. Those who have only a slight knowledge of the mathematical advances during the last sixty years know that a work written at the beginning of this period could not contain definitions of all the important terms now employed in mathematics.

On the other hand, it will probably be of interest to those readers who have